Laboratoire de mathématiques AGM UMR CNRS 8088 CY Cergy Paris Université 2, avenue Adolphe Chauvin

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France

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Employment

2023	Professor at CY Cergy Paris Université.
	Département de Mathématiques, Cergy-Pontoise, France.
$2022 - \dots$	Part-time Professor at École polytechnique.
	Département de Mathématiques, Palaiseau, France.
2020 - 2023	Researcher at CNRS (CR).
	Laboratoire de mathématiques AGM, CY Cergy Paris Université.
2015 - 2020	Researcher at CNRS (CR).
	Institut de Mathématiques de Bordeaux, Université de Bordeaux.
2013 - 2015	L.E. Dickson Instructor.
	Department of Mathematics, The University of Chicago. Contact: Carlos Kenig.
2010 - 2013	Graduate Student Instructor.
	Diderot University Paris 7 and Institut de Mathématiques de Jussieu.

Education

2022	Habilitation à Diriger des Recherches. CY Cergy Paris Université.
	Thesis defended on October 4, 2022.
	Referees/jury members: Patrick Gérard, Zhongwei Shen et Vladimír Šverák.
	Other jury members: Isabelle Gallagher, Thierry Gallay, David Gérard-Varet,
	Emmanuel Hebey, Pierre-Gilles Lemarié-Rieusset.
2010 - 2013	Ph.D. in Mathematics, summa cum laude. Diderot University Paris 7.
	Thesis defended on June 5, 2013.
	Supervisor: David Gérard-Varet. Referees: Carlos Kenig and Claude Le Bris.
2006 - 2010	Student at École Normale Supérieure of Rennes.
2010	M.Sc. in Mathematics, summa cum laude.
	Pierre and Marie Curie University Paris 6.
	Numerical Analysis and Partial Differential Equations.
2009	Agrégation in Mathematics.
	Highest competitive French state exam for the recruitment of college teachers.
2008	First year Master studies in Mathematics and Physics, summa cum laude.
	ENS of Rennes and Rennes 1 University.
2007	B.Sc. in Mathematics, summa cum laude. ENS of Rennes.
	B.Sc. in Physics, cum laude. Rennes 1 University.

Grants (PI)

2022 - 2024	EMERGENCE grant of 'CY Initiative' acronym CYFI (CYngular fluids and in-
	terfaces); co-PI Charles Collot; Awarded Amount: 45 000 euros.
2022 - 2023	Postdoctoral fellowship of Pedro Fernandez-Dalgo funded by the LABEX MME-
	DII; Awarded Amount 51 000 euros.
2022 - 2023	Postdoctoral fellowship of Jin Tan funded by the LABEX MME-DII; Awarded
	Amount 51 000 euros.

2017 - 2022	PI ANR-16-CE40-0027-01 BORDS (Boundaries, Oscillations, layeRs in Differen-
	tial Systems) funded by the ANR; Awarded Amount: 124 200 euros.
2016 - 2019	PI "Junior Chair" BOLIDE from the IDEX University of Bordeaux; Awarded
	Amount: 108 000 euros.
2016	PI PEPS "Young Researcher" of the CNRS; Awarded Amount: 3 000 euros.
2015 - 2018	PI NSF Grant DMS 1500893: Asymptotic Analysis of Partial Differential Equa-
	tions and Systems with Emphasis on Boundary Layers; Awarded Amount: 130
	000 dollars.

Other grants

2023 - 2026	Member of the collaborative project BOURGEONS funded by the ANR; PI Anne-
	Laure Dalibard.
2021 - 2025	Member of the collaborative project CYNA funded by the 'CY Initiative
	d'Excellence'; PI Philippe Gravejat.
2021 - 2024	Member of the collaborative project CRISIS funded by the ANR; PI Francesco
	Fanelli.
2018 - 2022	Member of the collaborative project SingFlows funded by the ANR; PI David
	Gérard-Varet.

Memberships

$2023 - \dots$	Member of the research group 'GDR Defis theoriques pour les sciences du climat',
	France.
$2023 - \dots$	Member of the research group 'GDR Navier-Stokes 2.00', France.
$2021 - \dots$	Member of the GAMM Activity Group Analysis of PDEs 'Gesellschaft für ange-
	wandte Mathematik und Mechanik', Germany.

Publications and preprints

- W. Jing, Y. Lu and C. Prange. Unified quantitative analysis of the Stokes equations in dilute perforated domains via layer potentials. To appear in *Multiscale Modeling and Simulation* (2025).
- C. Prange and J. Tan. Free boundary regularity of vacuum states for incompressible viscous flows in unbounded domains. Submitted (2023).
- T. Barker, P. Fernández-Dalgo and C. Prange. Blow-up of dynamically restricted critical norms near a potential Navier-Stokes singularity. *Mathematische Annalen*, 389, 1517-1543, (2024).
- T. Barker, C. Prange and J. Tan. On symmetry breaking for the Navier-Stokes equations. *Comm. Math. Phys.* 405(25), (2024).
- D. Albritton, T. Barker and C. Prange. Epsilon regularity for the Navier-Stokes equations via weak-strong uniqueness. *Journal of Mathematical Fluid Mechanics* 25(49) (2023); special issue in the memory of Olga Ladyzhenskaya.
- T. Barker and C. Prange. From concentration to quantitative regularity: a short survey of recent developments for the Navier-Stokes equations. *Vietnam Journal of Mathematics* (2023); special issue dedicated to Carlos Kenig's 70th birthday.
- D. Albritton, T. Barker and C. Prange. Localized smoothing and concentration for the Navier-Stokes equations in the half space. *Journal of Functional Analysis*, 284(1) (2023).
- M. Higaki, J. Zhuge and C. Prange. Large-scale regularity for the stationary Navier-Stokes equations over non-Lipschitz boundaries. *Analysis & PDE*, 17(1), 171-242 (2024).
- R. Höfer, F. Sueur and C. Prange. Motion of several slender rigid filaments in a Stokes flow. Journal de l'École polytechnique – Mathématiques, Tome 9, 327-380 (2022).

- T. Barker and C. Prange. Mild criticality breaking for the Navier-Stokes equations. *J. Math. Fluid Mech.*, 23, 66 (2021).
- E. Bocchi, F. Fanelli and C. Prange. Anisotropy and stratification effects in the dynamics of fast rotating compressible fluids. *Annales de l'Institut Henri Poincaré C, Analyse Non Linéaire*, 39(3), 647-704 (2022).
- T. Barker and C. Prange. Quantitative regularity for the Navier-Stokes equations via spatial concentration. *Comm. Math. Phys.* 385(2), 717-792 (2021).
- M. Higaki and C. Prange. Regularity for the stationary Navier-Stokes equations over bumpy boundaries and a local wall law. *Calc. Var. Partial Differential Equations*, 59, 131 (2020).
- T. Barker and C. Prange. Scale-invariant estimates and vorticity alignment for Navier-Stokes in the half-space with no-slip boundary conditions. *Arch. Ration. Mech. Anal.*, 235(2), 881-926 (2020).
- T. Barker and C. Prange. Localized smoothing for the Navier-Stokes equations and concentration of critical norms near singularities. *Arch. Ration. Mech. Anal.*, 236, 1487-1541 (2020).
- Y. Maekawa, H. Miura and C. Prange. On stability of blow-up solutions of the Burgers vortex type for the Navier-Stokes equations with a linear strain. *J. Math. Fluid Mech.*, 21(4), 46 (2019).
- Y. Maekawa, H. Miura and C. Prange. Local energy weak solutions for the Navier-Stokes equations in the half-space. *Comm. Math. Phys.*, 367(2), 517-580 (2019).
- Y. Maekawa, H. Miura and C. Prange. Estimates for the Navier-Stokes equations in the half-space for non localized data. *Analysis & PDE*, 13(4), 945-1010 (2020).
- C. Kenig and C. Prange. Improved regularity in bumpy Lipschitz domains. *J. Math. Pures Appl.*, 113, 1-36 (2018).
- S. Armstrong, T. Kuusi, J.-C. Mourrat and C. Prange. Quantitative analysis of boundary layer correctors in periodic homogenization. *Arch. Ration. Mech. Anal.*, 226(2), 695-741 (2017).
- C. Kenig and C. Prange. Uniform Lipschitz estimates in bumpy half-spaces. Arch. Ration. Mech. Anal., 216(3), 703-765 (2015).
- D. Bresch and C. Prange. Newtonian limit for weakly viscoelastic fluid flows. SIAM J. Math. Anal., 46(2), 1116-1159 (2014).
- A.-L. Dalibard and C. Prange. Well-posedness of the Stokes-Coriolis system in a half-space over a rough surface. *Analysis & PDE*, 7-6, 1253-1315 (2014).
- C. Prange. Asymptotic analysis of boundary layer correctors in periodic homogenization. SIAM J. Math. Anal., 45(1), 345-387 (2013).
- C. Prange. First-order expansion for the Dirichlet eigenvalues of an elliptic system with oscillating coefficients. Asymptot. Anal., 83(3), 207-235 (2013).

Proceedings

- C. Prange. Large-scale regularity for fluids over rough boundaries. Oberwolfach Reports, (2021) 10.14760/OWR-2021-14.
- C. Prange. Infinite energy solutions to the Navier-Stokes equations in the half-space and applications. Séminaire Laurent Schwartz-EDP et applications, Exp. No. 2, 18 p., 2017–2018.
- C. Prange. Uniform estimates in homogenization: compactness methods and applications. Journées Équations aux Dérivées Partielles, Exp. No. 7, 25 p., 2014.

Supervision

2021 – 2025 Moustapha Agne, first **master 2 student**, then **Ph.D. student**, CY Cergy Paris University. He is the sole author of the following preprint 'Asymptotic Analysis of Boundary Layers for Stokes Systems in Periodic Homogenization' https://arxiv.org/abs/2407.11841.

2021 - 2024	Jin Tan, postdoctoral researcher , CY Cergy Paris University. I was his postdoc
	mentor. From September 1st, 2024, he is research assistant professor at The
	Chinese University of Hong-Kong, Hong-Kong.
2021 - 2023	Pedro Fernández-Dalgo, postdoctoral researcher , CY Cergy Paris University.
	I was his co-mentor with Tobias Barker. From October 1st, 2024, he is a
	postdoctoral fellow at the Basque Center for Applied Mathematics, Spain.
2020	Richard Höfer, postdoctoral researcher , Institut de Mathématiques de Bor-
	deaux, France. I was his co-mentor with Franck Sueur. From January 1st,
	2023, he is associate professor at the University of Regensburg, Germany.
2019	Mitsuo Higaki, postdoctoral researcher, Institut de Mathématiques de Bor-
	deaux, France. I was his postdoc mentor. From October 1st, 2019, he is
	associate professor at Kobe University, Japan.
2016 - 2019	Edoardo Bocchi, Ph.D. student , Institut de Mathématiques de Bordeaux,
	France. He graduated from the University of Bordeaux on September 23,
	2019. I co-supervised his Ph.D. studies with David Lannes. From September
	1st, 2023, he is a Marie Skłodowska-Curie postdoctoral researcher in the group
	of Filippo Gazzola at Politecnico of Milan, Italy.

Conference talks

2024	▷ CIRM, Luminy, France. Mathematics of fluids in motion: Recent results and trends.
	▷ Sevilla, Spain. ECM2024, mini-symposium 'Current developments in mathematical
	fluid dynamics'.
2023	▶ Münster, Germany. Stability, mixing and fluid dynamics.
	▶ Lyon, France. Aspects locaux et non-locaux en mécanique des fluides.
2022	▷ CIRM, Luminy, France. Mathflows 2022.
	▷ ICMAT, Madrid, Spain. Recent trends in fluid mechanics.
	▷ Lyon, France. Summer School on Fluid and Turbulence.
	▷ New York University, Abu Dhabi. SITE conference, 'Long Time Behavior and Sin-
	gularity Formation in PDEs - Part V' (online).
	▷ Euler International Mathematical Institute, StPetersburg, Russia. 'Mathematical
	Hydrodynamics: the Legacy of Olga Ladyzhenskaya and Modern Perspectives' in
	honor of the 100th anniversary of Olga Ladyzhenskaya (postponed).
	▷ Kyoto University, Japan. KTGU Mathematics Workshop for Young Researchers
	(online).
2021	▷ Universidad de Cantabria, Spain. TURB1D 2021 (online participation).
	▷ University of Freiburg, Germany. 9th GAMM-Seminar on Analysis of PDEs.
	▷ University Paris-Saclay. Workshop 'Analyse Harmonique et EDP'.
	▷ Oberwolfach workshop (online). Homogenization Theory: Periodic and Beyond.
2020	▷ SIAM TXLA annual meeting (online). Topics in qualitative and quantitative properties of partial differential equations.
	▶ Madrid, Spain. Workshop on PDEs. May 2020. Cancelled due to the Covid-19 pandemic.

	▷ Kyoto University, Japan. Kyoto Top Global University workshop. Participation
	cancelled due to the Covid-19 pandemic.
2019	▷ Lyon, France. Franco-Brazilian meeting in mathematical fluid mechanics.
	▷ Cardiff, UK. Small Scales and Homogenisation.
2018	▷ Evry, France. Kolmogorov Days.
	▷ Porquerolles, France. Mathflows 2018.
	▷ Durham, UK. Homogenisation in Disordered Media.
2017	▷ Erlangen, Germany. SIAM Conference on Mathematical and Computational Issues
	in the Geosciences.
	▷ Saint-Etienne de Tinée, France. Dynamics and PDEs.
2016	▷ University of Bordeaux, France. Second French-Korean Conference in Mathematics.
2015	▷ RIMS, Kyoto University, Japan. Asymptotic Problems for Differential Equations and
	Viscosity Solutions.
	▷ Banff International Research Station, Banff, Canada. Developments in the theory of
	Homogenization.
2014	▷ Mittag-Leffler Institute, Stockholm, Sweden. Homogenization and Random Phe-
	nomenon.
	▷ Journées EDP, Roscoff, France.
2013	▷ AMS Fall Southeastern Sectional Meeting, University of Louisville, USA.

Seminar talks

2023	▷ The Hong Kong Polytechnic University. Analysis seminar (online); 2 talks.
	▶ Hong Kong University of Science and Technology and Beijing Normal University,
	China. Joint PDE seminar (online).
	▷ University of Evry, France. Analysis seminar.
	▷ IHÉS, France. X-IHÉS seminar.
2022	▷ University of Kentucky, US. Analysis and PDE seminar (online).
2021	▷ Princeton University, US. Fluids seminar (online).
2020	▷ University of Bath, UK. Asymptotics, Operators, and Functionals (online); 2 talks.
	▷ University of Bordeaux, France. PDE Seminar.
	▷ Département de Mathématiques d'Orsay, France. Séminaire Analyse et EDP (online).
	⊳ Fields Institute, Toronto, Canada. Hydrodynamics Seminar (online).
	▷ UAM-UC-ICMAT-IMUS Analysis and PDEs Seminar, Spain (online).
	▷ IMUS, Seville, Spain. Fluid Talks.
2019	▷ LAGA, Université Paris 13, France. Nonlinear PDE Seminar.
2018	▷ Max Planck Institute MIS, Leipzig, Germany. Uni-MPI Oberseminar.
	▷ University of Oxford, UK. PDE lunchtime seminar.
2017	▷ Tokyo Institute of Technology, Japan. PDE Seminar. December 2017.
	▷ University of Toulouse, France. MIP Seminar.
	▷ IHÉS, Paris, France. X-EDP Seminar.
	▷ ULB, Brussels, Belgium. Nonlinear Analysis and PDE Seminar.
	▷ Collège de France, Paris, France. PDE Seminar.
2016	▷ Aalto University, Finland. Analysis and Geometry Seminar.
	▷ University of Nice Sophia Antipolis, France. SingWave Analysis Seminar.
	⊳ The University of Chicago, USA. Calderón-Zygmund Analysis Seminar.

	▷ Tokyo Institute of Technology, Japan. PDE Seminar.
	▷ University of Paris Est, France. PDE Seminar.
	▷ University of Cergy-Pontoise, France. Geometry and PDE Seminar.
2015	▷ University of Hiroshima, Japan. Applied Mathematics Seminar.
	▷ University of Lyon, France. Applied Mathematics Seminar.
	▷ University of Bordeaux, France. PDE Seminar.
	▷ Institut de Mathématiques de Bordeaux, France. Department Day.
	▷ Nečas Center for Mathematical Modeling, Prague, Czech Republic.
	▷ University of Illinois, Chicago, USA. March 2015.
	▶ The University of Chicago, USA. Calderón-Zygmund Analysis Seminar.
2014	▷ University of Cambridge, UK. GAPDE Seminar.
	▷ University of Kentucky, USA.
2013	▶ The University of Chicago, USA. CAMP/PDE Seminar.
	▶ École Normale Supérieure, Rennes, France. Analysis and PDE Seminar.
	▷ Institut Henri Poincaré, Paris, France. Analysis and PDE Seminar.
2012	▷ LAMA, University of Savoy, Chambéry, France. Analysis Seminar.

Invited Lectures

- Doctoral course (6 hours) at Indian Institute of Technology Bombay. February 2025.
 - 'Quantitative methods in homogenization: regularity, concentration and renormalization'
- ▷ **Doctoral course** (4 hours) at Kyoto University (Kyoto Top Global University Special Lectures), Japan. December 2017.
 - 'Blow-up, compactness and (partial) regularity in Partial Differential Equations'

Organization of scientific events

2020	▷ Co-organizer of the Analysis Seminar of the AGM laboratory at CY Cergy Paris University.
2020 - 2025	▷ Co-organizer of the Colloquium of the AGM laboratory at CY Cergy Paris University.
2025	▷ Co-organizer of the conference 'CY Days in Nonlinear Analysis' at CY Cergy Paris University.
2024	▷ Co-organizer with Charles Collot of the summer school 'Summer school: (in)-stability phenomena in fluid mechanics'. CY Advanced Studies.
2023	▷ Co-organizer of the thematic day 'Kinetic equations in Cergy' at CY Cergy Paris University.
2022	▷ Co-organizer of the inaugural conference of the CYNA project at CY Cergy Paris University.
2021	▷ Co-organizer of the thematic day 'Boundary layers in Cergy' at CY Cergy Paris University.
2020	▷ Co-organizer with Hao Jia of a three-days workshop at the University of Cergy-Pontoise "Workshop on nonlinear fluid and dispersive PDEs". This event is cancelled due to the Covid-19 pandemic.
2018 - 2020	▷ Organizer of the Colloquium of the AGM laboratory at the University of Cergy-Pontoise.
2017	▷ Co-organizer of the conference in honor of Jean-Claude Saut's 70th birthday at the University of Bordeaux.
2016 - 2018	▷ Organizer of a seminar at the University of Bordeaux in an original format: one hour and a half mini-course in the morning and one hour standard seminar in the afternoon by the same speaker. I organized 7 sessions.

(ENPC) and postdocs at The University of Chicago.

▷ Organizer of a panel discussion between students of the French École des Ponts

> Participation in the organization of the scientific and innovation contest

Professiona	"C.Génial 2013" in Paris.
2024	▷ Deputy director of the AGM research center, CNRS and CY Cergy Paris University.
	▶ Head of the new Master's program in Mathematics for AI, Machine Learning and Scientific Computing (MACIA: Mathématiques pour l'Apprentissage, le Calcul et l'Intelligence Artificielle), CY Cergy Paris University.
	▶ Tutor of two trainees of the engineering school CY Tech (Thales and Safran).
	▶ Member of the hiring committee for temporary teaching and research professors (ATER) at CY Cergy Paris University.
2023 –	▶ Teaching coordinator and head of the first year, Master's program in mathematics, CY Cergy Paris University.
	▶ Member of AGM's laboratory council.
	${\bf \triangleright} \ Member \ of \ the \ scientific \ council \ and \ local \ point \ of \ COFUND \ MathPhdInFrance.$
$2022-\dots$	▷ Local contact point at the AGM laboratory for environmental issues.
2025	 Head of a hiring committee for a professor (PR) at CY Cergy Paris University. Member of a hiring committee for an assistant professor (MCF) at Paris-Saclay University.
2024	Referee and jury member for the Ph.D. thesis of David Llerena at the University of Paris-Saclay.
2023	 Jury member for the Ph.D. thesis of Elena Salguero at ICMAT & IMUS, Spain. Member of a hiring committee for an assistant professor (MCF) at ENS Lyon. Referee and jury member for the Ph.D. thesis of Nacer Aarach at Université de Bordeaux.
2022	Dordon D
	▶ Member of the research group 'GDR Labos 1point5' https://labos1point5. org on environmental issues in academia and public research. The main objective is to bring our community to meeting the objectives of the Paris Agreement.
2021	▷ Committee member for the Ph.D. thesis of Pedro Fernández-Dalgo at the University of Paris-Saclay.
2019	Committee member for the Ph.D. thesis of Marco Bravin at the University of Bordeaux.
2018	▶ Member of the board of examiners for the entrance exams to the École Normale

Teaching at CY Cergy Paris University and CY Tech

2024 – 2025 Statistics (54 hours).

 $2014 - \dots$

2015

2013

Master 1 course. Lectures and tutorials.

Topology and functional analysis (54 hours).

Supérieure (Lyon, Paris-Saclay, Rennes, Ulm).

ENS, ARMA, CMP, CPAM, JEMS, JMPA.

▷ Referee for peer-reviewed journals among which Inventiones Math., Ann. Sci.

Master 1 course. Lectures and tutorials.

	Graduate course (9 hours).
	Refresher course in mathematical analysis and probability. Lectures.
2023 - 2024	Statistics (54 hours).
	Master 1 course. Lectures and tutorials.
	Advanced differential calculus and differential geometry (36 hours).
	Master 1 course. Lectures.
	Linear optimization (9 hours).
	First-year engineering course (third-year undergraduate studies). Lectures.
	Probability (16 hours).
	First-year undergraduate studies. Lectures and tutorials.
	Graduate course (9 hours).
	Refresher course in mathematical analysis and probability. Lectures.
2022 - 2023	Graduate course on PDEs (30 hours).
	'Regularity for Partial Differential Equations: elliptic equations, homogenization
	and fluid mechanics'.
	Graduate course (9 hours).
	Preparation for the competitive examination 'Agrégation'.
	Graduate course (10 hours).
	Refresher course in mathematical analysis.
2021 - 2022	Graduate course on PDEs (30 hours).
2020 - 2021	Graduate course on PDEs (30 hours; online).
	Master 1 course (18 hours; online).
	Reading group based on "Differential Equations, Dynamical Systems, and an In-
	troduction to Chaos" by Hirsch, Smale and Devaney and "Les Matrices" by
	Serre.
2019 - 2020	Graduate course on PDEs (20 hours).
2018 - 2019	Graduate course on PDEs (30 hours).
2017 - 2018	Graduate course on PDEs (30 hours).

Teaching at École polytechnique

2024 - 2025	Functional Analysis (MAT 452) at École polytechnique (40 hours).
	Second-year course (first-year of graduate studies). Tutorials.
2023 - 2024	Introduction to Real Analysis (MAT 361) at École polytechnique (36 hours).
	First-year course (third-year undergraduate studies). Tutorials.
	Functional Analysis (MAT 452) at École polytechnique (40 hours).
	Second-year course (first-year of graduate studies). Tutorials.
2022 - 2023	Introduction to Real Analysis (MAT 361) at École polytechnique (36 hours).
	Functional Analysis (MAT 452) at École polytechnique (20 hours).

Previous teaching

2016	Graduate course at the University of Bordeaux (12 hours).
	'Weak and strong convergence methods for Partial Differential Equations'.
2013 - 2015	Instructor (200 hours). The University of Chicago.
Summer 2015	Introduction to PDEs. 4.5 hours course in the REU (Research Experience for
	Undergraduates) of The University of Chicago.
Winter 2015	Math 273, Basic Theory of Ordinary Differential Equations.
Fall 2014	Math 200, Mathematical Methods for Physical Science 1.
	Math 204, Analysis 2: differential calculus in \mathbb{R}^n .

Winter 2014	Math 205, Mathematical Methods for Physical Science 2.
	Math 201, Analysis 3: multiple integrals and theorems of Green, Gauss, Stokes.
Fall 2013	Math 200, Mathematical Methods for Physical Science 1.
	Math 204, Analysis 2: differential calculus in \mathbb{R}^n .
2010 - 2013	Teaching Assistant at undergraduate level. Diderot University Paris 7.
Spring 2010	Pierre and Marie Curie University Paris 6. Problem sessions.
2009 - 2010	Oral examiner in preparatory classes to elite schools.

Other service

2020	Elected parents' representative at my children's school.
$2019 - \dots$	Elected member of my apartment building residents' committee
	(Conseil Syndical).

Languages

 $\textbf{French}: \ \text{Native.} \ \textbf{German}: \ \text{Mother tongue.} \ \textbf{English}: \ \text{Excellent.} \ \textbf{Swedish}: \ \text{Good.}$